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Numerical solution of radiative transfer within a cubic enclosure filled with nongray gases using the WSGGM

Won-Hee Park[1,*] and Tae-Kuk Kim[2]

[1]*Railroad Environment Research Department, Korea Railroad Research Institute, 360-1, Woram-dong, Uiwang-si, Gyeongi-do Korea*
[2]*School of Mechanical Engineering, Chung-Ang University, 221 Heukseok-dong, Dongjak-gu Seoul Korea*

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Abstract

Using a weighted sum of gray gases model with gray gas regrouping technique (WSGGM-RG), problems of radiative transfer within three-dimensional enclosures filled with non-homogeneous and non-isothermal combustion gas mixtures were solved. The radiative transfer equation was solved based on discrete ordinate method (DOM) and ray-tracing method (RTM) with WSGGM-RG. The results using WSGGM-RG were compared with those using the statistical narrow band (SNB) model. The average radiative intensities and radiative heat fluxes using WSGGM-RG corresponded relatively well to those using the SNB model as reference data. A very good computational efficiency was also noted.

Keywords: Radiation; WSGGM (Weighted sum of gray gases model); SNB (Statistical narrow band) model; DOM (Discrete Ordinate Method); RTM (Ray Tracing Method)

1. Introduction

Solution schemes for the radiative transfer and modeling of radiation properties for molecular gases are required in analyzing the phenomena of radiative heat transfer in media whose absorption and emission characteristics vary depending on the wavelength, e.g., combustion gas. Radiation properties for molecular gases are functions of wavelength, temperature, partial pressure and so on. As a relatively simple and accurate model, the weighted sum of gray gases model (WSGGM) was proposed by Hottel and Sarofim [1]. Modest [2], in particular, proved that this method was easily applicable to any arbitrary radiative heat transfer equation solvers. The modeling results of Smith et al. [3] are widely used, and Kim and Song [4, 5] applied WSGGM to narrow bands. WSGGM was improved in these narrow bands to make it applicable to gas mixtures containing arbitrary substances by Park and Kim [6]. Gray gas regrouping [7], which was also attempted to shorten the calculation time, produced good results. Although several solution methods are suggested for three-dimensional enclosures, the applicability and accuracy of these methods have yet to be tested fully, especially for nongray gases. This is because little measured data and/or reference solutions for three-dimensional nongray gas radiation problems can be found in the literature, such as results of Liu [8], Park and Kim [9], and Coelho [10].

In this paper, ray tracing method (RTM) and discrete ordinate method (DOM) are used to analyze the radiative heat transfer equation, and WSGGM with gray gas regrouping (WSGGM-RG) is used to predict the radiative properties for nongray gases. The calculation results were verified by comparing them with those obtained by Park and Kim [9], and a correspondence was noted, moreover, shortening the CPU time.

*Corresponding author. Tel.: +82 31 460 5358, Fax.: +82 31 460 5319
E-mail address: whpark@krri.re.kr
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2. Weighted sum of gray gases model with gray gases regrouping (WSGGM-RG)

The equation for the absorption coefficient suggested by Kim and Song [4,5] and Park and Kim [7] expressed the local gray gas absorption coefficient and total weighting factor of CO_2-H_2O mixtures as:

$$\kappa_{mix} = \kappa_1 \frac{e^{\alpha_1/T}}{T^2} PX_{CO_2} + \kappa_2 \frac{e^{\alpha_2/T}}{T^2} PX_{HO_2} . \quad (1)$$

Six κ_1, κ_2 and five α_1, α_2 are considered. With the combination of six κ_1, κ_2 and five α_1, α_2, the total number of gray gases required for a mixture gas is 900. The detailed process of acquiring the parameters of WSGGM is explained in Kim and Song's paper [4]. To improve the computational efficiency of WSGGM for gas mixtures while maintaining accuracy, Park and Kim [7] suggested WSGGM-RG wherein the original number of gray gases was regrouped into a designated number (N_G). As the new weighting factor, W_i is obtained by simply summing up the original weighting factors of gray gases in the i-th group as:

$$W_i = \sum^{N_i} W_{mix} , \quad (2)$$

where N_i denotes the number of gray gases in the i-th group and W_i is a function of the gas temperature, partial pressure of CO_2 and H_2O and the temperature reference (T_{ref}), partial pressure reference of CO_2 ($P_{CO2,ref}$)and H_2O ($P_{H2O,ref}$) where the reference values are used for the gray gas regrouping process.

The new absorption coefficient of the i-th group (κ_i) can be obtained by using an equation similar to the Planck mean absorption coefficient:

$$\kappa_i = \sum^{N_i} \kappa_{mix} \Big/ W_i \quad (3)$$

Physically, the weighting factor in Eq. (2) corresponds to the fraction of blackbody energy in the spectral region of the effective absorption coefficient.

3. Solvers for the radiative transfer equation

3.1 Ray tracing method (RTM)

The radiative heat transfer equation for the distance coordinates in media that absorb, emit, and do not scatter is expressed as:

$$\frac{dI}{ds} = \kappa I_b - \kappa I , \quad (4)$$

where I and κ refer to the radiative intensity and absorption coefficient, respectively; the lower subscript b refers to the blackbody. To use WSGGM, radiative heat transfer equation (4) can be expressed in WSGGM form as:

$$\frac{dI_i}{ds} = \kappa_i W_i I_b - \kappa_i I_i , \quad (5)$$

where κ_i refers to the i-th gray gas, and, W_i, to the weighting factor of the gray gas representing the fraction of blackbody radiation energy accounted for by the gray gas of κ_i.

Considering the line-of-sight of the $\vec{\Omega}$ direction in Fig. 1 wherein radiation energy is transferred in the media in terms of the isothermal and homogeneous grids, the radiative heat transfer equation expressed by using WSGGM in Eq. (5) can be discretized as:

$$\begin{aligned} I_{i,j+1}(\vec{\Omega}) &= I_{i,j}(\vec{\Omega})\exp\left[-\kappa_{j+1/2}(s_{j+1}-s_j)\right] \\ &\quad + W_{i,j+1/2} I_b \exp\left[1-\exp\left\{-\kappa_{j+1/2}(s_{j+1}-s_j)\right\}\right] \end{aligned} \quad (6)$$

where s_j denotes the distance in the line-of-sight of $\vec{\Omega}$ direction to the j-th grid point. The i-th radiative intensity on the walls is calculated as:

$$I_{i,0} = W_i I_{b,0} = W_i \varepsilon_0 \sigma T_0^4 , \quad (7)$$

where ε_0 and s refer to the emissivity on the walls and Stefan-Boltzmann constant, respectively. Since every wall is a blackbody, ε_0=1.0. All radiative intensities for i-th gray gas in the $\vec{\Omega}_m$ direction can be obtained by using Eqs. (6, 7), and the radiative inten-

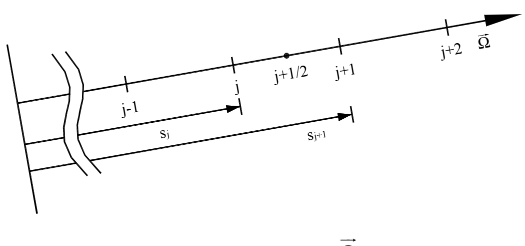

Fig. 1. Grids in the line-of-sight of the $\vec{\Omega}$ direction.

sity in the $\vec{\Omega}_m$ direction at point j, by using the following equation:

$$I_j(\vec{\Omega}) = \sum_{i=1}^{N_G} I_{i,j}(\vec{\Omega}) , \tag{8}$$

where N_G refers to the number of gray gas groups in WSGGM. Eq. (6) should be solved for as many as N_G number of gray gas groups. The radiative heat flux and average radiative intensity can be made dimensionless as follows:

$$q_0^* = \frac{q_0}{\sigma T_{max}^4} = \frac{1}{\sigma T_{max}^4} \int_{\Omega=4\pi} (\hat{n}\cdot\vec{\Omega})\, I_0(\vec{\Omega}) d\Omega \approx \frac{1}{\sigma T_{max}^4} \sum_{m=1}^{N_O} (\hat{n}\cdot\vec{\Omega}_m) I_0(\Omega_m)\Delta\Omega_m \tag{9}$$

$$G_j^* = \frac{G_j}{\sigma T_{max}^4/\pi} = \frac{1}{\sigma T_{max}^4/\pi} \int_{\Omega=4\pi} I_j(\Omega) d\Omega \approx \frac{1}{\sigma T_{max}^4/\pi} \sum_{m=1}^{N_O} I_j(\Omega_m)\Delta\Omega_m \tag{10}$$

where T_{max} denotes the maximum temperature for making them dimensionless, and $\hat{n}$, the unit normal vector on the wall. The lower subscript m symbolizes the discretization for the discrete directions by Thurgood et al. [11] adopted to determine the discrete directions and their angular weights, whereas N_o refers to the total number of discrete ordinates. This study used T_{60} in the T_N quadrature, with N_o as the total number of discrete directions to 28,800. To compare with the reference data directly, RTM-T_{60} was chosen because the reference data is obtained by using RTM-T_{60}. It is helpful whether the WSGGM-RG is well done in 3-dimensional enclosure by using the same RTE solver and quadratures as those used for the reference data. In terms of heat flux, the direction of radiative heat flux toward the media from the wall was set to positive. Eq. (6) was calculated by modifying the program of Park and Kim [9] by using RTM.

3.2 Discrete ordinates method (DOM)

Considering the radiative heat transfer equation in media that absorb, emit, and do not diffuse by dividing it into N_o discrete directions in a three-dimensional x-y-z space coordinate, Eq. (3) for the m-th discrete direction can be estimated as:

$$\left[\omega_{x,m}\frac{\partial}{\partial x} + \omega_{y,m}\frac{\partial}{\partial y} + \omega_{z,m}\frac{\partial}{\partial z} + \kappa_i\right] I_{m,i} = \kappa_i W_i I_b . \tag{11}$$

The program of Kim [12] was modified for analyzing the above WSGGM-formed radiative transfer equation. For DOM T_4 (128 discrete directions) was employed. The dimensionless radiative heat flux and average radiative intensity can be obtained by using Equations (9) and (10).

4. Numerical results

4.1 Conditions for numerical analysis

Park and Kim [9] found accurate solutions for radiative transfer for a black-walled cubic enclosure filled with nongray gas mixtures having non-uniform temperature and non-homogeneous concentration profiles. RTM was used for the solution, with the T_{60} quadrature set (28,800 discrete directions). Transmittances through the medium with non-uniform temperature and concentration profiles are then computed by RADCAL [13] with Curtis-Godson approximation [14]. Their solutions are considered for the reference data in this study. Analyses were performed under the same system and conditions assumed by Park and Kim [9]. The cube of L × L × L (L= 1m) was considered as shown in Fig. 2, of which all the walls were blackbody. Three different temperature distributions, wherein the temperature of the media and that of the walls and partial pressures for each gas change along the z-axis, were also considered. In each case, gases were composed of combinations of CO_2, H_2O, and N_2; the partial pressure of each gas was 1 atmospheric pressure ($P_{CO2}+P_{H2O}+P_{N2}$ = 1.0atm). Fig. 3 shows the profiles of partial pressure CO_2 and temperature along z-axis. The cases in Fig. 3 are summarized in Table 2.

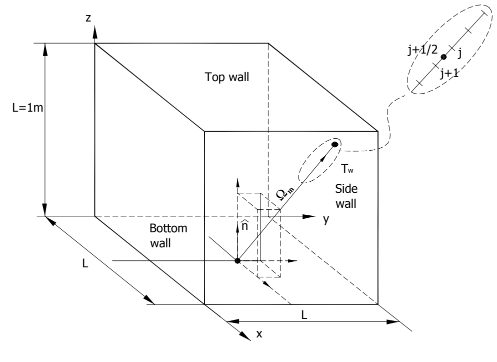

Fig. 2. Schematic drawing of a cubic enclosure.

Table 1. Profiles of temperature and partial gas pressure considered for the 3-D cubic system.

Case	Temperature (K)	Mole fraction
1	$z=1$; T_{max} $z \neq 1$; T_{min}	$X_{CO_2}=1.0$
2	$T_{min}+(T_{max}-T_{min})z$	$X_{CO_2}=z$, $X_{CO_2}=1-z$
3	$z \le 0.5$; $1500-3000\left(\frac{0.5-z}{0.5}\right)^2$ $+2000\left(\frac{0.5-z}{0.5}\right)^3$ $0.5 \le z \le 1$; $1500-900\left(\frac{z-0.5}{0.5}\right)^2$ $+600\left(\frac{z-0.5}{0.5}\right)^3$	$z \le 0.5$; $X_{CO_2}=\frac{1}{3}-\left(\frac{0.5-z}{0.5}\right)^2$ $+\frac{2}{3}\left(\frac{0.5-z}{0.5}\right)^3$, $X_{H_2O}=2X_{CO_2}$ $0.5 \le z \le 1$ $X_{H_2O}=2/3$, $X_{CO_2}=1/3$

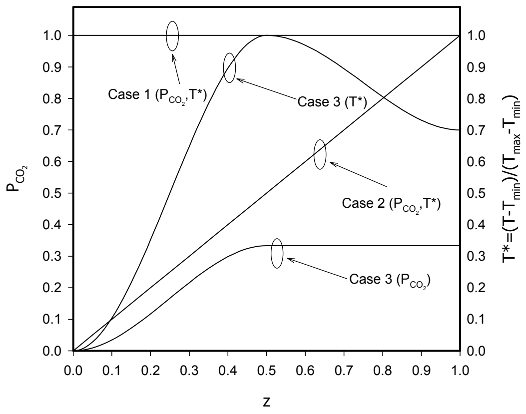

Fig. 3. Profiles of concentration and temperature along the z axis.

4.2 Comparison of regrouped gray gas numbers

The reference values for gray gas regrouping were set as follows: T_{ref}= 500 K, $P_{CO2,ref}$= 1.0, and $P_{H2O,ref}$= 0.0 for Case 1; T_{ref} = 1000 K, $P_{CO2,ref}$ = 0.5, $P_{H2O,ref}$ = 0.5 for Case 2, and; T_{ref} = 1000 K, $P_{CO2,ref}$= 0.2, and $P_{H2O,ref}$ =0.4 for Case 3.

Tables 2-4 show the average and maximum of the relative errors obtained using RTM-T_{60}/ DOM-T_4 and WSGGM with 3, 5, 7, 10, 15, and 20 regrouped gray gases in each case. Average of relative errors is obtained as

$$\left(\text{Relative Error}\right)_{\text{Ave}} = \frac{1}{\text{J}}\sum_{j=1}^{\text{J}} Q_{j,\text{WSGG}} \big/ Q_{j,\text{SNB}} \tag{12}$$

Table 2. Average and maximum relative errors (%) for various numbers of regrouped gray gases in Case 1 as obtained using RTM-T_{60} and DOM-T_4.

Location (m)			(x, 0.5, 0)	(x, 0.5, 1)	(0.5, 1, z)	(0.5, 0.5, z)
WSGGM -3RG	RTM-T_{60}	Ave.	12.00	0.0	9.26	7.57
		Max.	12.15	0.0	12.34	9.72
	DOM-T_4	Ave.	12.50	0.13	10.13	9.29
		Max.	15.27	0.13	13.23	12.46
WSGGM -5RG	RTM-T_{60}	Ave.	6.67	0.0	5.85	5.09
		Max.	6.69	0.0	6.73	5.56
	DOM-T_4	Ave.	7.20	0.13	6.79	6.91
		Max.	10.19	0.13	11.46	9.39
WSGGM -7RG	RTM-T_{60}	Ave.	6.66	0.0	5.85	5.09
		Max.	6.69	0.0	6.73	5.56
	DOM-T_4	Ave.	7.20	0.13	6.79	6.91
		Max.	10.19	0.13	11.46	9.39
WSGGM -10RG	RTM-T_{60}	Ave.	6.61	0.0	5.82	5.07
		Max.	6.63	0.0	6.66	5.53
	DOM-T_4	Ave.	7.14	0.13	6.76	6.89
		Max.	10.14	0.13	11.45	9.36
WSGGM -15RG	RTM-T_{60}	Ave.	6.43	0.0	5.62	4.88
		Max.	6.46	0.0	6.50	5.34
	DOM-T_4	Ave.	6.96	0.13	6.56	6.70
		Max.	9.97	0.13	11.28	9.17
WSGGM -20RG	RTM-T_{60}	Ave.	6.43	0.0	5.62	4.88
		Max.	6.46	0.0	6.50	5.34
	DOM-T_4	Ave.	6.96	0.13	6.56	6.70
		Max.	9.97	0.13	11.28	9.17

Table 3. Average and maximum relative errors (%) for various numbers of regrouped gray gases in Case 2 as obtained using RTM-T_{60} and DOM-T_4.

Location (m)			(x, 0.5, 0)	(x, 0.5, 1)	(0.5, 1, z)	(0.5, 0.5, z)
WSGGM -3RG	RTM-T_{60}	Ave.	5.12	7.04	5.74	0.92
		Max.	5.14	7.18	29.53	3.76
	DOM-T_4	Ave.	3.58	7.10	6.69	1.11
		Max.	5.87	7.76	37.96	4.64
WSGGM -5RG	RTM-T_{60}	Ave.	6.21	2.38	5.62	2.34
		Max.	6.60	2.46	7.66	5.07
	DOM-T_4	Ave.	7.38	2.58	5.54	2.18
		Max.	9.47	3.17	11.97	4.14
WSGGM -7RG	RTM-T_{60}	Ave.	7.04	2.38	5.46	2.40
		Max.	7.47	2.47	7.72	5.63
	DOM-T_4	Ave.	8.22	2.58	5.05	2.23
		Max.	10.33	3.17	8.12	4.38
WSGGM -10RG	RTM-T_{60}	Ave.	7.67	0.94	6.37	2.60
		Max.	8.13	1.11	8.17	6.07
	DOM-T_4	Ave.	8.82	1.38	4.36	2.38
		Max.	10.95	1.68	8.20	4.72
WSGGM -15RG	RTM-T_{60}	Ave.	7.95	0.59	6.69	2.71
		Max.	8.43	0.78	8.41	6.26
	DOM-T_4	Ave.	9.12	1.09	4.49	2.47
		Max.	11.25	1.55	8.50	4.89
WSGGM -20RG	RTM-T_{60}	Ave.	7.95	1.09	6.86	2.82
		Max.	8.45	1.13	8.56	6.24
	DOM-T_4	Ave.	9.15	1.15	5.00	2.56
		Max.	11.30	3.61	8.53	4.92

Table 4. Average and maximum relative errors (%) for various numbers of regrouped gray gases in Case 3 as obtained using RTM-T_{60} and DOM-T_4.

Location (m)			(x, 0.5, 0)	(x, 0.5, 1)	(0.5, 1, z)*	(0.5, 0.5, z)
WSGGM -3RG	RTM-T_{60}	Ave.	3.95	15.86	10.53	2.73
		Max.	4.06	23.18	26.25	4.66
	DOM-T_4	Ave.	2.51	18.45	10.98	3.04
		Max.	3.91	38.13	22.12	5.91
WSGGM -5RG	RTM-T_{60}	Ave.	5.07	2.15	2.04	1.14
		Max.	5.14	2.73	5.18	4.63
	DOM-T_4	Ave.	3.33	6.17	6.32	1.89
		Max.	5.13	12.46	20.41	4.72
WSGGM -7RG	RTM-T_{60}	Ave.	5.15	3.11	2.16	1.09
		Max.	5.25	4.15	5.21	4.65
	DOM-T_4	Ave.	3.38	6.21	6.22	1.87
		Max.	5.21	11.06	22.02	4.74
WSGGM -10RG	RTM-T_{60}	Ave.	5.10	3.15	2.12	1.01
		Max.	5.21	4.29	5.04	4.57
	DOM-T_4	Ave.	3.35	6.22	6.07	1.79
		Max.	5.18	10.91	22.29	4.72
WSGGM -15RG	RTM-T_{60}	Ave.	5.42	5.56	3.34	1.09
		Max.	5.59	7.59	7.68	4.77
	DOM-T_4	Ave.	3.59	6.51	6.48	1.83
		Max.	5.51	12.81	26.08	4.89
WSGGM -20RG	RTM-T_{60}	Ave.	2.93	1.79	1.83	0.64
		Max.	3.05	2.60	3.93	2.70
	DOM-T_4	Ave.	1.69	6.19	4.96	1.18
		Max.	2.96	12.90	20.65	3.11

* Since the reference value at z = 0.85m was very small for (0.5m, 1m, z), the error at z = 0.85m was not considered for the maximum and average errors.

where J is 19 which is the calculated points along the (x, 0.5m , 0m), (x, 0.5m, 1m), (0.5m, 1m, z) and (0.5m, 0.5m, 0.5m), $Q_{j,\mathrm{WSGG}}$ is obtained by using WSGGM-RG in this work, and $Q_{j,\mathrm{SNB}}$ is reference data [9]. Since the RTM used in this case employed the same T_{60} as are reference data, the relative errors shown in Tables 2-4 for RTM-T60 can be compared separately from the errors obtained using WSGGM-RG in three-dimensional enclosure except the error components arising from the different solvers of analyzing the radiative transfer equation. For RTM-T_{60}, in Case 1, note that the average and maximum values of relative errors in all locations being compared decreased with the increasing number of regrouped gray gases (Table 2). Likewise, in Cases 2 and 3, the average and maximum values tended to decrease as the number of regrouped gray gases increased. As seen in most cases, the calculation results obtained using RTM-T_{60} corresponded better to the reference data compared to those obtained using DOM-T_4. In all cases wherein RTM-T_{60} was used and five or more gray gases were regrouped, the average value of relative errors was approximately less than 8%; the maximum of relative errors was approximately less than 8.5%. Note that the difference of average and maximum values is small at all the points compared. For (0.5m, 1m, z) in Case 3, the values used as reference in the location (0.5m, 1m, 0.85m) were very small; the values of the calculated relative errors were relatively large, however. To prevent such overestimation of errors, the location (0.5m, 1m, 0.85m) was excluded in the calculation of the average and maximum values of the relative errors (Tables 4). As the calculation results obtained with WSGGM-5RG, the errors were observed to decrease greatly compared with the errors obtained by WSGGM-3RG. Moreover, even if the number of regrouped gray gases was increased more than 5 gray gases, the accuracy did not improve greatly. In the dimensionless radiative heat flux of (x, 0.5m, 1m) for Case 1, the top wall was hot; as such, other walls where line-of-sights started and the media was line-of-sight passed through were cold for Case 1. In this case, dimensionless radiative fluxes were calculated only by the integrals for solid angles regardless of the modeling of radiation properties. Therefore, they were smaller than the errors of the resulting values in any other locations. The relative errors of the dimensionless radiative heat flux in Cases 1 and 2 along the location (x, 0.5m, 0m) on the lower wall where the temperature is low are larger than considered locations for almost number of regrouped gray gas groups shown as Table 2 and 3. In all cases wherein five or more gray gases were regrouped, the average value of relative errors was approximately less than 9%; the maximum of relative errors was approximately less than 26%, with high relative errors in x = 0.85 - 0.95m in the (x, 0.5m, 0m) location of Case 3 when DOM-T_4 is used.

4.3 Comparison of results using DOM-T_4 and RTM-T_{60}

To assess the sensitivity of the solutions according to the number of regrouped gray gases, the average and maximum values of relative errors according to the number of regrouped gray gases were compared in the preceding section. Since accuracy did not improve significantly even if five or more gray gases were used, WSGGM-5RG was selected in all cases for purposes of comparison.

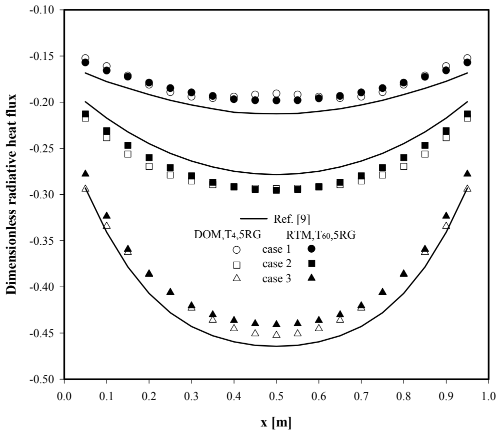

Fig. 4. Comparison of the dimensionless radiative bottom wall heat fluxes obtained by different methods.

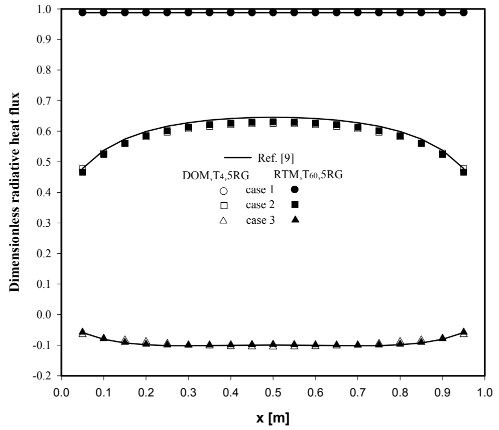

Fig. 5. Comparison of the dimensionless radiative top wall heat fluxes obtained by different methods.

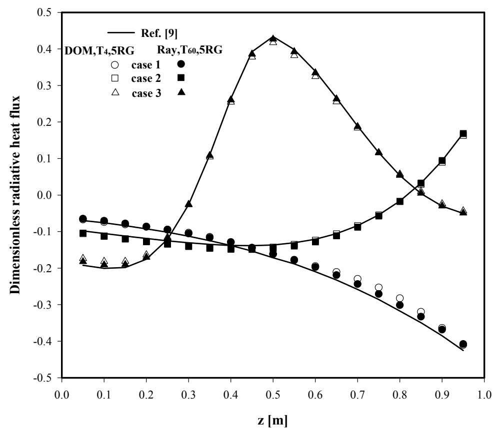

Fig. 6. Comparison of the dimensionless side wall heat fluxes obtained by different methods.

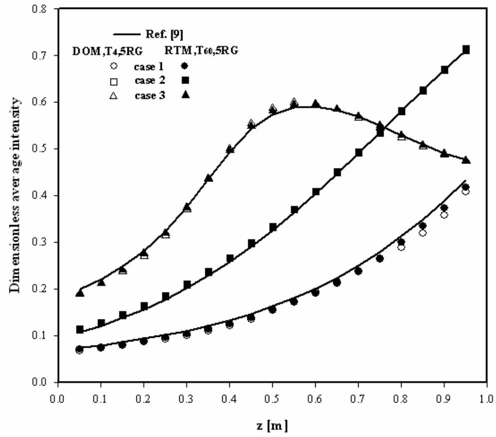

Fig. 7. Comparison of the dimensionless average radiative intensities obtained by using different methods.

The resulting values are shown in each location for all cases in one figure. Fig. 4 compares the reference data and results obtained by using RTM with WSGGM-5RG and results obtained using DOM with WSGGM-5RG in relation to the dimensionless radiative heat flux in the bottom wall (x, 0.5m, 0m). Figs. 5, 6, and 7 show the dimensionless radiative heat flux values on the top wall (x, 0.5m, 1m), dimensionless average radiative heat fluxes on the side wall (0.5m, 1m, z), and dimensionless average radiative intensities along the (0.5m, 0.5m, z), respectively. Note that the solid symbols denote the results obtained by using RTM, and the empty symbols, the results obtained with DOM. In Case 1, there were major differences between the reference values and dimensionless radiative heat flux values in the bottom wall regardless of whether the RTM or DOM was used (Fig. 4). The average and maximum values of relative errors in Case 1 were approximately less than 6.7% when the RTM-T_{60} was used. In contrast, the average value of relative errors was approximately less than 7.2%; the maximum value was 11.4% when DOM-T_4 was used, so that the relative errors were not large. As shown in Fig. 4, the greatest dimensionless radiative heat flux value was not found in the location of x = 0.5m; instead, the highest value of error occurred in the location of x = 0.35m, 0.65m for the results of dimensionless radiative heat flux on the bottom wall where low temperature was maintained by DOM. This in turn could be attributed to the typical ray effects [15] arising from DOM. In contrast, such errors did not occur when RTM was used. For the dimensionless top radiative wall, the heat flux obtained by using

DOM and WSGGM-5RG was that arising from the difference between radiative transfer solvers. For Case 1, the top wall radiative heat fluxes both for RTM and DOM almost coincided with reference l, as shown in Fig. 5. In Case 2, the average and maximum of relative errors when RTM was used in all the compared locations were approximately less than 6.2% with the maximum value at about 7.7%, but the average value of relative errors was about less than 7.4%, with the maximum value at about 12.0%. For the dimensionless radiative heat flux on the top wall, several smaller errors occurred near the location of x = 0.2m with the use of RTM, unlike when DOM was used; the opposite results occurred near the location of x = 0.5m. For dimensionless radiative heat flux of the side wall in Fig. 6, there were larger differences found between the reference values and resulting values obtained by using DOM in the location of z=0.65-0.85m than those between the reference values and results obtained with RTM. In case 2 as shown in Fig. 7, dimensionless average radiative intensities along the center of the system (0.5m, 0.5m, z), both the dimensionless average radiative intensity obtained using RTM and that obtained using DOM corresponded well with the reference solution. Similar to the cases of the dimensionless radiative side wall heat flux and average radiative intensity in Case 2, the results obtained with DOM were sometimes found to be more accurate than those obtained by using RTM, since errors arising from the approximation of DOM offset those arising from WSGGM-5RG, hence the results that were closer to the reference solution. Likewise, in Case 3, which considered media of non-uniform temperature and partial pressure distributions, very satisfactory results were obtained; the average error was below 6.3% in all the result values regardless of the analysis method used. Compared with the computation time taken with a computer equipped with a 2.4GHz CPU, it took 1244 minutes to perform the calculations using the RTM-T_{60} calculating the radiation properties with the SNB model, which was the same numerical procedure as obtaining the reference results [9]. Using WSGGM-5RG and RTM-T_{60} took 10.8 minutes, thereby shortening the calculation time to about 1/115 compared to the case of using the SNB model. Moreover, calculation using DOM employing T_4 together with WSGGM-5RG took only about 21 seconds. Therefore, WSGGM-RG together with DOM considered in this study is effective for predicting the radiative characteristics of real gas mixtures in a three-dimensional enclosure well.

5. Conclusion

In this study, we analyzed the radiative heat transfer characteristics in a three-dimensional cube filled with gas mixtures of H_2O, CO_2, and N_2. The characteristics of the radiative properties of gas mixtures were studied by using WSGGM-RG. For the analysis of the radiative heat transfer equations, DOM and RTM were used as the method of solution. T_4 (128 discrete directions) was employed in the case of DOM, and T_{60} (28,800 discrete directions), in the case of RTM. Analyses were performed in three cases, with the resulting values compared with the previous fairly accurate results [9]. WSGGM-RG considered in this study is effective for predicting the radiative characteristics of real gas mixtures in a three-dimensional enclosure well. WSGGM-RG together with DOM can be used to calculate fast for accurate results from an engineering perspective.

Nomenclature

G	:	Average radiative intensity, W/m^2 sr
I	:	Radiative intensity, $W/m^2 sr$
N_G	:	Number of gray gases
N_O	:	Number of discrete ordinates
P	:	Total pressure, atm
q	:	Radiative heat flux, W/m^2
s	:	Distance in the line-of-sight, m
T	:	Temperature, K
X	:	Mole fraction
x,y,z	:	Coordinate, m
W	:	Weighting factor for WSGGM

Greek symbols

α_1, α_2	:	Parameters of absorption coefficient for WSGGM
ε	:	Emissivity
κ	:	Absorption coefficient, 1/m
κ_1, κ_2	:	Parameters of absorption coefficient for WSGGM
σ	:	Stefan-Boltzmann constant, $5.67\times10^{-8} W/m^2K^4$
Ω	:	Direction of line-of-sight
ω	:	Direction cosine

Subscripts

0 : Wall
b : Blackbody
est : Estimation
i : I-th gray gas
j : J-th grid
m : M-th ordinate
max : Maximum value
min : Minimum value
mix : H_2O-CO_2 mixture gas
ref : Reference

Superscripts

* : Dimensionless

References

[1] H. C. Hottel and A. F. Sarofim, Radiative Transfer, McGraw-Hill, New York, 1967.

[2] M. F. Modest, Weighted-sum-of-gray-gases model for arbitrary solution methods in radiative transfer, *J. Heat Trans.-T. ASME,* 113 (3) (1991) 650-656.

[3] T. F. Smith, Z. F. Shen and J. N. Friedman, Evaluation of coefficients for the weighted sum of gray gases model, *J. Heat Trans.-T. ASME,* 104 (4) (1982) 602-608.

[4] O. J. Kim and T. H. Song, Implementation of the weighted sum of gray gases model on a narrow band: application and validity, *Numer. Heat Tr. B-Fund.,* 30 (4) (1996) 453-468.

[5] O. J. Kim and T. H. Song, Database of WSGGM-based spectral methods for the radiation of combustion products, *J. Quant. Spectrosc. Ra.,* 64 (4) (2000) 379-394.

[6] W. H. Park and T. K. Kim, Application of the weighted sum of gray gases model for non-homogeneous gas mixtures with arbitrary compositions, Proceedings of Eurotherm 73 on Computational Thermal Radiation in Participating Media, Mons, Belgium, April 15-17 (2003) 129-137.

[7] W. H. Park and T. K. Kim, Development of WSGGM using a gray gas regrouping technique for the radiative solution within a 3-D enclosure filled with non-uniform gas mixtures, *JSME Int. J. B-Fluid T.,* 48 (2) (2005) 310-315.

[8] F. Liu, Numerical solutions of three-dimensional non-gray gas radiative transfer using the statistical narrow band model, *J. Heat Trans.-T. ASME.,* 121 (2) (1999) 200-203.

[9] W. H. Park and T. K. Kim, Narrow band solutions of radiative transfer within a cubic enclosure filled with real gas mixtures. *KSME Int. J.,* 15 (6) (2002) 861-869.

[10] P. J. Coelho, Numerical simulation of radiative heat transfer from non-gray gases in three-dimensional enclosures. *J. Quant. Spectrosc. Ra.,* 74 (3) (2002) 307-328.

[11] C. P. Thurgood, A. Pollard and H. A. Becker, The TN quadrature set for the discrete ordinates method, *J. Heat Trans.-T. ASME.,* 117 (4) (1995) 1068-1070.

[12] T. K. Kim, CCRHT-3D (Combined Conduction-Convection-Radiation Heat Transfer in Three Dimensions) program, Chung-Ang Univ., 1999.

[13] W. L. Grosshandler, Radcal: a narrow-band model for radiation calculations in a combustion environment, Technical Report NIST Technical Note 1402, National Institute Standard and Technology, 1993.

[14] W. L. Godson, The evaluation of infrared radiation fluxes due to atmospheric water vapor, *Quart. J. Royal Meteorological Society,* 79 (1953) 367-379.

[15] P. J. Coelho, Role of ray effects and false scattering on the accuracy of standard and modified discrete ordinate methods, *J. Quant. Spectrosc. Ra.,* 73 (2-5) (2002) 231-238.